Small clusters of fermions

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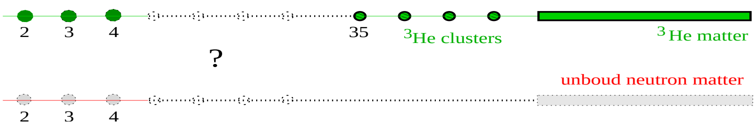
Several theoretical studies [1] indicate that there is no any reasonable chance for 3n and 4n to be bound. GANIL experiment [2] suggesting tetraneutron has not been confirmed but these no-result had however the merit of pushing theoreticians at work to clarify the possible existence of small n-clusters. If the situation seems clear for A=3 and 4, it is less well established for a larger number of neutrons. The search of multi-neutron resonances raises also some interest and a recent experiment scanning the 4n continuum in the d(^{8}He,^{6} Li)4n reaction reports a 2-3 MeV width structure [3].

In this issue it is enlightening to make a parallel with a similar, better-known, fermionic system: the ^{3}He atomic clusters. Since small ^{3}He droplets can exist [4], should we expect some stability islands (see figure below) in the continuum neutron states going from N=2 to N=∞. If yes, where? If not, why?



The two-body interactions of these systems look at first glance quite similar. We have compared in Fig. 1 the n-n AV18 [5] and the ^{3}He-^{3}He Aziz [6] S-wave potentials. They have been rescaled by the corresponding masses $\frac{M}{\hbar^2}$ and the resulting length units, as well as the inter-particle distance d are fm and $\mathring{A}$ respectively. Corresponding low energy parameters are given in Table 1. In the n-n case we have also considered Reid93 [7] and MT13 [8] potentials, the latter being adjusted to reproduce the experimental scattering length. None of these system supports a bound dimer ($a < 0$) but ^{3}He seems less favorable

Table 1
Low energy n-n and ^{3}He-^{3}He parameters.

	n-n(fm)				He-He($\mathring{A}$)
	Av18	Reid	MT13	Exp.	Aziz 91
a	-18.49	-17.54	-18.59	-18.59 ± 0.4	-7.24
r_0		2.85	2.94	2.75 ± 0.1	13.5
η_c	1.08	1.09	1.10		1.30

to make clusters. This can be seen by calculating the critical values of the scaling factor η_c introduced in the potential $V^{(\eta)}(r) = \eta V_{nn}(r)$, which bounds a dimer. For n-n, this value is around $\eta_c = 1.08$ whereas for He-He is sensibly greater $\eta_c = 1.30$ (see Table 1).

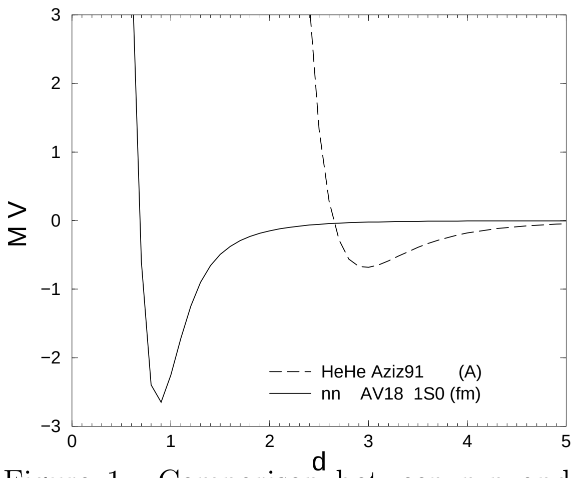


Figure 1. Comparison between n-n and ^{3}He-^{3}He potentials.

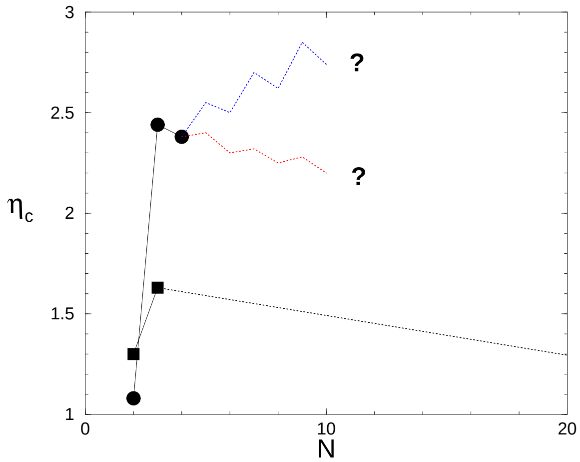


Figure 2. Critical value of the scaling factor for n (circles) and ^{3}He (squares)

When examining larger systems we first consider the bosonic case. Despite the absence of dimers, "bosonic" neutron trimers and tetramers do exist with binding energies $B_{n_3} \approx$ 1 MeV and $B_{n_4} \approx$ 10 MeV. This is however not the case for atomic ^{3}He, suggesting once more that neutron clusters should be favored with respect to atomic ^{3}He. They all disappear when the Pauli principle is imposed, but can reappear – as in ^{3}He – if the number of interacting particles is increased. The existence of small clusters results thus from a compromise between an attractive pairwise interaction and the effective Pauli repulsion.

In order to study such a balance, we have investigated in several directions [9]: *(i)* scaling factor in V_{nn}, *(ii)* three-neutron interactions (TnI), *(iii)* influence of n-n P- waves *(iv)* confining the system in an harmonic oscillator (HO) trap and *(iv)* "dimer"-"dimer" scattering. It is always possible to bind 3- and 4-n states by modifying the usual n-n and/or TnI models but the violation has to be very strong, producing serious anomalies. Thus, in case *(i)* one needs a large scaling factor $\eta_c \sim 3$. Using ad-hoc TnI, one gets a very compact object in which NN force becomes even repulsive. Keeping the usual S-wave n-n potential, the enhancement of n-n P-waves required to bound a 3- or 4-n system is such that they become themselves resonant!

We have confined N=2,3,4 n in an HO trap with fixed frequency ω and size parameter $b = \sqrt{\frac{\hbar}{m\omega}}$. HO is the only external field in which "internal" and "center of mass" energies can be properly separated. In absence of n-n forces the "internal" energies are known analytically but can be obtained as well by solving the N-body "internal" problem, with pairwise HO potential of frequency $\left(\frac{\omega}{\sqrt{N}}\right)$. The effect of n-n interaction has been evaluated by solving the internal problem with

$$V_{ij} = \frac{1}{2}\, m \, \left(\frac{\omega}{\sqrt{N}}\right)^2 \, r_{ij}^2 + V_{nn}(r_{ij})$$

and calculating the difference between the pure HO energy and the $HO+V_{nn}$ one: $B_N = E_{HO}^{(N)} - E_{HO+nn}^{(N)}$.

Results concerning the ground state are given in Table 2 for several values of b. Some comments are in order: *(i)* there is a clear indication of paring effect when going from

Table 2
Binding energies B_N of N neutrons in an HO trap with size parameter b.

		b=2				b=3				b=4			
N	J^π	$E_{HO}^{(N)}$	B_N	$\frac{B_N}{N}$	$\frac{B_N}{E_{HO}}$	$E_{HO}^{(N)}$	B_N	$\frac{B_N}{N}$	$\frac{B_N}{E_{HO}}$	$E_{HO}^{(N)}$	B_N	$\frac{B_N}{N}$	$\frac{B_N}{E_{HO}}$
2	0^+	15.55	6.34	3.17	0.41	6.91	3.13	1.56	0.45	3.89	1.81	0.93	0.47
3	$\frac{3}{2}^-$	41.47	9.74	3.25	0.23	18.43	4.41	1.47	0.24	10.36	2.55	0.85	0.25
4	0^+	67.39	15.30	3.58	0.23	29.95	7.40	1.69	0.25	16.82	4.31	1.08	0.26

N=2 $\rightarrow$ 3 $\rightarrow$ 4 *(ii)* one has always $B_4 > 2B_2$, suggesting an effective attraction between dineutrons *(iii)* the binding energy per particle increases when going from N=2 to N=4 (iv) the ratio $\frac{B_N}{E_{HO}}$ tends to a constant value independent of b. The preceding results tend to indicate that there is a benefit when going from 2 to 4. The main difference in respect to ^{3}He has been found in the role of P-waves. Their influence in n case is attractive but very small whereas they significantly contribute to the ^{3}He binding energy ($\sim 40\%$). The reason for such a different behaviour is the hard core radius of the corresponding potentials, which differ by a factor 3 (see Fig. 1). The centrifugal barrier is one order of magnitude smaller in ^{3}He and the effective potential is, contrary to n case, still attractive in regions where it can play a role. This difference can be dramatic in binding larger fermion systems.

Despite the negative results quoted in [1], the question of larger neutron clusters merits some attention. At present, the strongest argument against their existence are the mean-field results, all concluding to an unbounded infinite nuclear matter but it should be possible to reach the same conclusion "from below", i.e. from a systematic study of few-neutron systems. This can be performed by studying the N-dependence of the critical scaling factor η_c, calculated simultaneously for n and ^{3}He. Our technology allow us to reach only N=4 with the results displayed on Fig. 2. The value $\eta_c^{(n)}$ makes a large jump when passing from N=2 to N=3 but starts to decreases when going from N=3 to N=4. Is that a pure numerical accident or, as in ^{3}He, an indication of a descent towards the $\eta = 1$ axis?. More powerful methods could go far beyond N=4 and draw a definite conclusion on this problem.

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